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# AN LU IMPLICIT SCHEME FOR HIGH SPEED INLET ANALYSIS

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## ABSTRACT

A numerical method is developed to analyze the inviscid flowfield of a high speed inlet by the solution of the Euler equations. The LU implicit scheme in conjunction with adaptive dissipation proves to be an efficient and robust nonoscillatory shock capturing technique for high Mach number flows as well as for transonic flows.

## INTRODUCTION

Recent interest in the aerospace plane and other hypersonic vehicles revitalized the research on high speed propulsion systems as well as hypersonic aerodynamics. In the design of supersonic and hypersonic propulsion systems the analysis of high speed flow past an inlet plays a critical role. While there are half a dozen propulsion study concepts for high speed flight, many candidate concepts share a common idea of combination of turboramjet engines for sub and supersonic flights and scramjet (supersonic combustion ramjet) engines for hypersonic flight. Resulting speeds of flows through the engines range widely from subsonic to hypersonic regimes.

The analysis of hypersonic flows would require the full Navier-Stokes equations with slip effects and chemical reaction. It is also important to

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understand the complex structure of shock waves. The turboramjet inlet flowfield includes the incoming supersonic flow deflected by oblique shock waves and the subsonic diffuser flow after the terminal normal shock wave while the scramjet inlet flow is characterized by strong oblique shock waves. The Euler equations which represent hyperbolic conservation law can be a useful testbed for developing and evaluating a shock capturing numerical algorithm.

It has been a difficult task for computational aerodynamicists to capture nonoscillatory shock waves as a converged solution. Unbounded growth of spurious oscillations often resulted in numerical instability. It is well known that upwind difference schemes can eliminate oscillations in the neighborhood of shock waves at the expense of a substantial increase of computational work. In parallel with the developments in upwind schemes it has been found that steady aerodynamic flows containing moderately strong shock waves can be quite well predicted by a central difference scheme augmented by a carefully controlled blend of first and third order dissipative terms.<sup>1</sup> In this paper the performance of adaptive dissipation is demonstrated for strong oblique shock waves in high Mach number flows on a near-uniform mesh.

Although a space marching method has been useful, it is not well-posed when there is upstream influence through subsonic portions of the flowfield such as a boundary layer. It also cannot handle the terminal shock and the subsonic diffuser flowfield as well as the flow with streamwise separation. Early time-integration codes for calculating the supersonic flow through an inlet used popular MacCormack schemes.<sup>2,3</sup> A disadvantage of these schemes for steady state calculation is that the computed steady state depends on the time step. Another drawback of MacCormack's implicit scheme is the difficulty in treating boundary conditions.

During the last decade, the Navier-Stokes equations have been the subject of exploratory investigations aimed at establishing the feasibility of their

solution, but the methods so far developed have been too expensive to permit their use in a routine production mode. However, recent developments of modern implicit schemes<sup>4-6</sup> in conjunction with multigrid methods are encouraging. The authors<sup>5</sup> developed an optimal dissipation model for the alternating direction implicit (ADI) scheme and proved that the improved ADI scheme is ideal for multigrid in two dimensions. Unfortunately, the ADI scheme in delta form to ensure the time step independent solution has stability and convergence problems in three dimensions. Two new implicit schemes which are unconditionally stable in any number of space dimensions were successfully developed by the authors<sup>6</sup> recently. They are lower-upper (LU) implicit scheme and LU-SSOR (symmetric successive over-relaxation) scheme. The LU implicit scheme has been tried on an H-mesh in this work for high speed flow calculations.

#### GOVERNING EQUATIONS

The Euler equations are obtained from the Navier-Stokes equations by neglecting viscous terms. Let  $\rho$ ,  $u$ ,  $v$ ,  $E$ ,  $H$ , and  $p$  be the density, Cartesian velocity components, total energy, total enthalpy, and pressure, and let  $x$  and  $y$  be Cartesian coordinates. Then for a two-dimensional flow these equations can be written as

$$\frac{\partial W}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \quad (1)$$

where  $W$  is the vector of dependent variables, and  $F$  and  $G$  are convective flux vectors

$$\begin{aligned} W &= (\rho, \rho u, \rho v, \rho E)^T \\ F &= (\rho u, \rho u^2 + p, \rho v u, u(\rho E + p))^T \\ G &= (\rho v, \rho u v, \rho v^2 + p, v(\rho E + p))^T \end{aligned} \quad (2)$$

The pressure is obtained from the equation of state

$$p = p(\gamma - 1) \left\{ E - \frac{1}{2} (u^2 + v^2) \right\} \quad (3)$$

These equations are to be solved for a steady state  $\partial W / \partial t = 0$  where  $t$  denotes time.

#### SEMI-DISCRETE FINITE VOLUME METHOD

A convenient way to assure a steady state solution independent of the time step is to separate the space and time discretization procedures. In semi-discrete finite volume method one begins by applying a semi-discretization in which only the spatial derivatives are approximated.<sup>1</sup> The use of a finite volume method for space discretization allows one to handle arbitrary geometries and helps one to avoid problems with metric singularities that are usually associated with finite difference methods. The scheme reduces to a central difference scheme on a Cartesian grid, and is second order accurate in space provided that the mesh is smooth enough. It also has the property that uniform flow is an exact solution of the difference equations.

#### NONLINEAR ADAPTIVE DISSIPATION

In typical calculation of flow with discontinuities by a central difference scheme, wiggles appear in the neighborhood of shock waves where pressure gradient is severe. In order to suppress the tendency for spurious odd and even point oscillations, and to prevent unsightly overshoots near shock waves, the scheme is augmented by artificial dissipative terms. The dissipative term, which is constructed so that it is of third order in smooth regions of the flow, is explicitly added to the residual. For the density equation, for example, the dissipation has the form

$$d_{i+1/2,j} - d_{i-1/2,j} + d_{i,j+1/2} - d_{i,j-1/2}$$

where

$$d_{i+1/2,j} = \epsilon_{i+1/2,j}^{(2)} (\rho_{i+1,j} - \rho_{i,j}) - \epsilon_{i+1/2,j}^{(4)} (\rho_{i+2,j} - 3\rho_{i+1,j} + 3\rho_{i,j} - \rho_{i-1,j}) \quad (4)$$

Let  $S$  be the cell area which is equivalent to the inverse of the determinant of transformation Jacobian. Both coefficients include a normalizing factor  $S_{i+1/2,j}/\Delta t$  proportional to the length of the cell side, and  $\epsilon_{i+1/2,j}^{(2)}$  is also made proportional to the normalized second difference of the pressure

$$v_{i,j} = \frac{|p_{i+1,j} - 2p_{i,j} + p_{i-1,j}|}{|p_{i+1,j} + 2p_{i,j} + p_{i-1,j}|} \quad (5)$$

in the adjacent cells. The third order terms provide background damping of high frequency modes. The first order terms are needed to control oscillations in the neighborhood of shock waves, and are turned on by sensing strong pressure gradients in the flow. The dissipative terms for the other equations are constructed from similar formulas with the exception of the energy equation where the differences are of  $\rho H$  rather than  $\rho E$ . The purpose of this is to allow a steady state solution for which  $H$  remains constant. Increasing the amount of artificial viscosity improves the rate of convergence although too much dissipation can hurt it. However, it is desirable to make the amount be as small as possible in order not to degrade the accuracy of solution. Typical amount of the third order terms is almost negligible when compared to the physical viscosity.

#### LU IMPLICIT SCHEME

Let the Jacobian matrices be

$$A = \frac{\partial F}{\partial W}, \quad B = \frac{\partial G}{\partial W} \quad (6)$$

and let the correction be

$$\delta W = W^{n+1} - W^n$$

here  $n$  denotes the time level.

The linearized implicit scheme for a system of nonlinear hyperbolic equations such as the Euler equations can be formulated as<sup>6</sup>

$$\{I + B \Delta t (D_x A + D_y B)\} \delta W + \Delta t R = 0 \quad (7)$$

where  $R$  is the residual

$$R = D_x F(W^n) + D_y G(W^n)$$

Here  $D_x$  and  $D_y$  are central difference operators that approximate  $\partial/\partial x$  and  $\partial/\partial y$ .

If  $\beta = 1/2$  the scheme remains second order accurate in time, for other values of  $\beta$ , the time accuracy drops to first order. The unfactored implicit scheme (Eq. (7)) produces a large block banded matrix which is very costly to invert and requires high storage. One can solve the system indirectly using a relaxation algorithm. Then it is desirable that the matrix should be diagonally dominant to meet a convergence criterion of a relaxation method. This can be achieved by flux splitting at the expense of a substantial increase in the computational work. Moreover, it seems that second order flux splitting methods in conjunction with relaxation algorithms are either conditionally stable or slow.

The operation count can be reduced by factorizing the operator of Eq. (7) approximately in various ways. The first way is known as the ADI scheme.<sup>7</sup>

$$(I + \beta \Delta t D_x A)(I + \beta \Delta t D_y B) \delta W + \Delta t R = 0 \quad (8)$$

Although the introduction of optimal artificial dissipation<sup>5</sup> makes the scheme be very desirable in two dimensions, the scheme in delta form is only conditionally stable in three dimensions. The ADI scheme introduces error terms of order  $(\Delta t)^3$  in three dimensions which reduce the convergence rate. If one concerns about memory requirement, each factor can be split into two subfactors.<sup>8</sup> If  $\beta = 1$ , the scheme becomes

$$(I + \Delta t D_x^- A^+)(I + \Delta t D_x^+ A^-)(I + \Delta t D_x^- B^+)(I + \Delta t D_x^+ B^-) \delta W + \Delta t R = 0 \quad (9)$$

where  $D_x^-$  and  $D_y^-$  are backward difference operators and  $D_x^+$  and  $D_y^+$  are forward difference operators. Each factor can be constructed using the diagonally dominant ADI factorization.<sup>9</sup> This scheme has six factors in three

dimensions and introduces error terms of order  $(\Delta t)^6$  which reduce the convergence rate further.

While the ADI scheme has been valuable in two dimensions, its inherent limitations in three dimensions suggest an alternative approach. An unconditionally stable implicit scheme which has error terms at most of order  $(\Delta t)^2$  in any number of space dimensions can be derived by the LU factorization.<sup>6</sup>

$$\{I + \beta \Delta t (D_x^- A^+ + D_y^- B^+)\} \{I + \beta \Delta t (D_x^+ A^- + D_y^+ B^-)\} \delta W + \Delta t R = 0 \quad (10)$$

Here,  $A^+$ ,  $A^-$ ,  $B^+$ , and  $B^-$  are constructed so that the eigenvalues of "+" matrices are nonnegative and those of "-" matrices are nonpositive.

$$\begin{aligned} A^+ &= \frac{1}{2} (A + r_A I), & A^- &= \frac{1}{2} (A - r_A I) \\ B^+ &= \frac{1}{2} (B + r_B I), & B^- &= \frac{1}{2} (B - r_B I) \end{aligned} \quad (11)$$

where

$$r_A > \max(|\lambda_A|), \quad r_B > \max(|\lambda_B|) \quad (12)$$

Here,  $\lambda_A$  and  $\lambda_B$  represent eigenvalues of Jacobian matrices. Equation (10) can be inverted in two steps. The LU implicit scheme needs the inversion of sparse triangular matrices which can be done efficiently without using large storage. This scheme has only two factors in three dimensions. Other forms of factorization in conjunction with flux splitting can be found in Ref. 10. For example,

$$\{I + \beta \Delta t (D_x^- A^+ + D_y^- B)\} \{I + \beta \Delta t D_x^+ A^- \delta W + \Delta t R = 0\} \quad (13)$$

This scheme requires a block-tridiagonal inversion in one direction. If one wants to include thin-layer viscous terms in the implicit operator, this scheme may be useful. However, it does not seem to be necessary to insert viscous terms into the operator when only the steady-state solution is desired.<sup>11</sup>



## RESULTS

In order to test the performance of the LU implicit scheme for high speed flow calculations, a two-dimensional model problem was selected. Figure 1 shows a typical hypersonic inlet and Fig. 2 shows a 54 by 32 H-mesh for a schematic high speed inlet. This mesh was used in all cases except the terminal shock wave problem where a 104 by 32 mesh was used for better resolution of the normal shock wave. However, all figures of convergence history are the results on the 54 by 32 mesh for comparison. The ramp angle is  $9^\circ$  and the shoulder angle is set to  $0.5^\circ$  for the terminal shock wave problem.

At the inflow boundary all the flow quantities are specified, and they are extrapolated from the interior at the outflow boundary for supersonic outflow. For the terminal shock wave problem the pressure was prescribed at the outflow boundary.

Four plots including Mach number contours, Mach number and pressure along the centerline, and the convergence history are shown in each set of Figs. 3 to 6. Terminal shock wave problem with freestream Mach number 2 is shown in Fig. 3. Figures 4 to 6 are for supersonic throughflows with freestream Mach numbers 5, 10, and 20, respectively. The pressure plots show the values of the pressure normalized by freestream static pressure so that the strengths of shock waves can be compared. Two indicators in the convergence histories are the maximum and the average density residual in logarithm scale.

As the figures show the wave structures in high Mach number flows including oblique shock waves, reflected shock waves, expansion fans, and the interaction of shock waves with expansion fans are successfully captured. These results clearly demonstrate the capability of the present numerical method for high speed flows. Figures 5 and 6 show that the location of the shock waves is hardly changed as the Mach number increases from 10 to 20.

However, the pressure plots show that the strengths of shock waves are quite different.

The convergence histories show that the residuals drop linearly and continuously. These prove the efficiency of the present numerical method. However, as the Mach number increases the convergence rate is slowed down. This problem will be investigated in the future. Another difficulty encountered in high Mach number flows is the sensitivity to the way of starting the solution procedure. Sudden introduction of the boundary condition to the freestream uniform flow is likely to cause numerical instability in high Mach number flows. Gradual increase of the time step is found to be effective to fix this problem.

#### CONCLUSION

The LU implicit scheme combined with the nonlinear adaptive dissipation is successfully developed as a robust and efficient shock capturing method for high Mach number inlet flows. It seems to be possible to improve the resolution and the accuracy of shock waves by using a total variation diminishing scheme. Extension to the Navier-Stokes equations is desirable for more accurate simulation of the flow through integrated high speed propulsion system.

#### ACKNOWLEDGMENT

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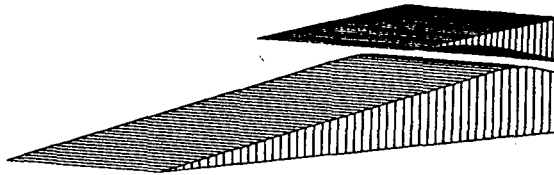


Figure 1. - A typical hypersonic inlet.

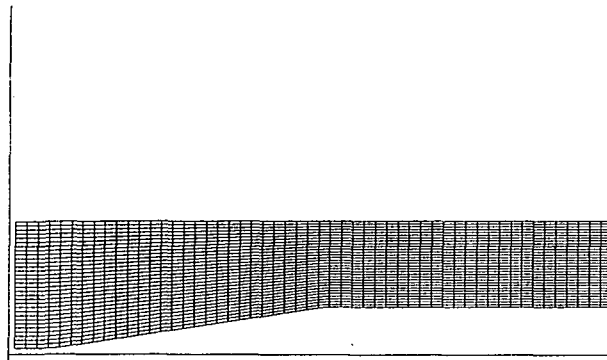
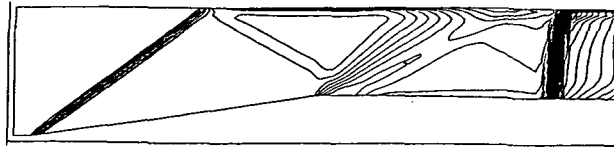
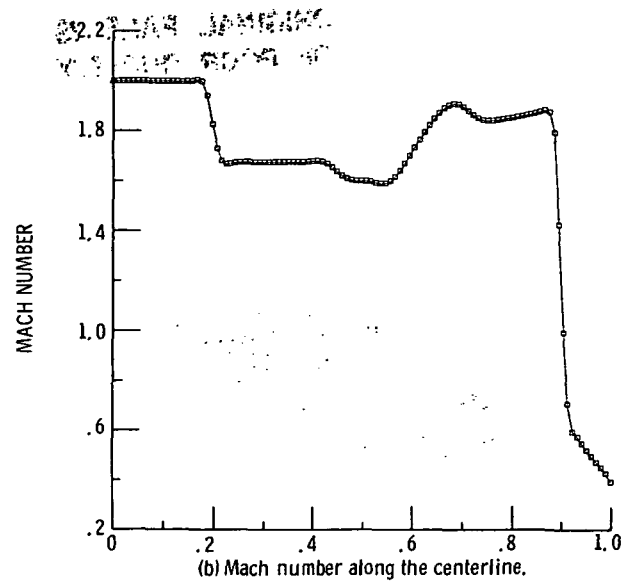


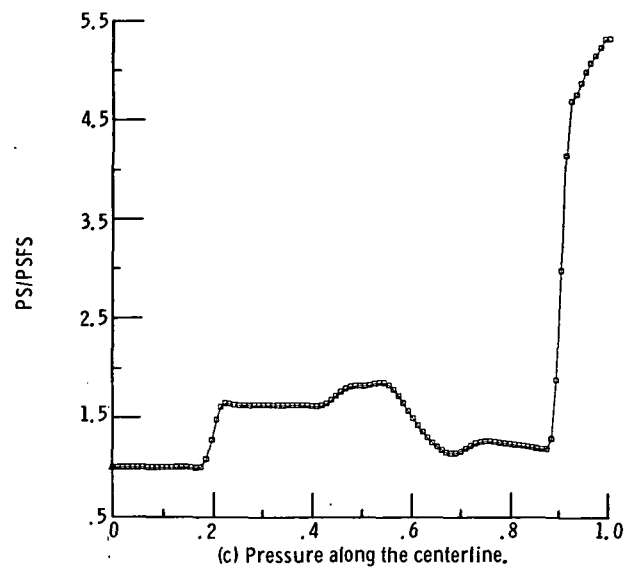
Figure 2. - 54x32 H-mesh for a schematic high speed inlet. Ramp angle  $90^\circ$  and shoulder angle  $0^\circ$  or  $0.5^\circ$ .



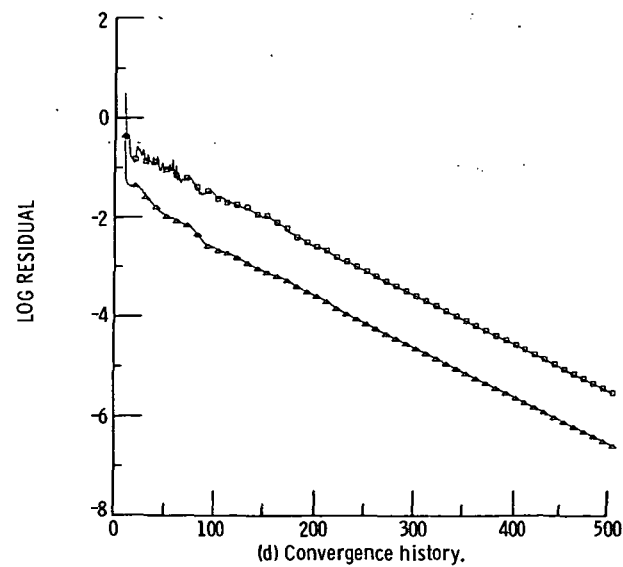
(a) Mach number contours.



(b) Mach number along the centerline.

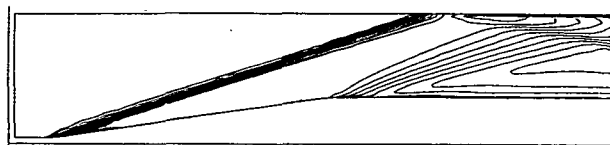


(c) Pressure along the centerline.

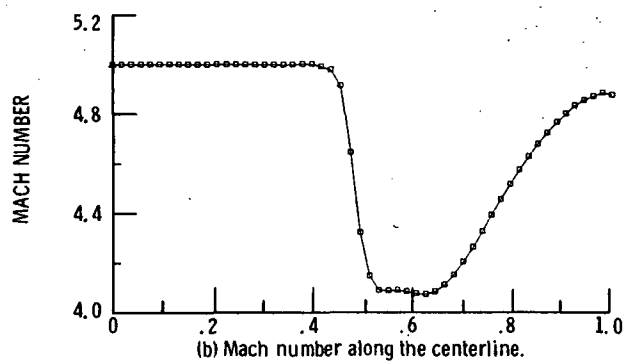


(d) Convergence history.

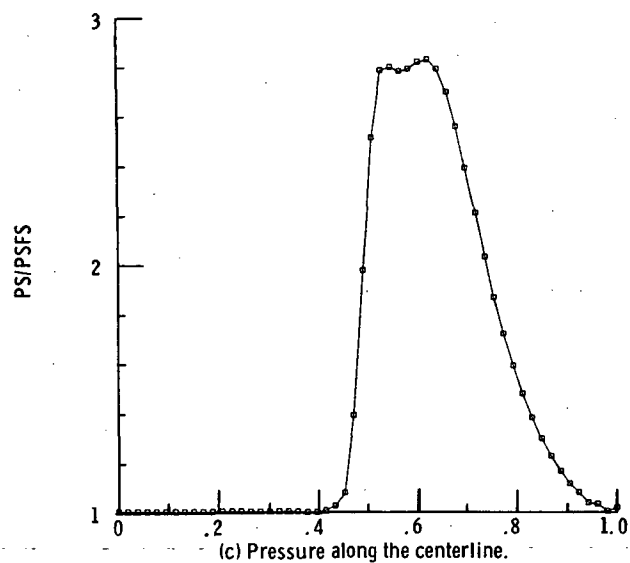
Figure 3. - Mach 2 inlet with the terminal shock wave.



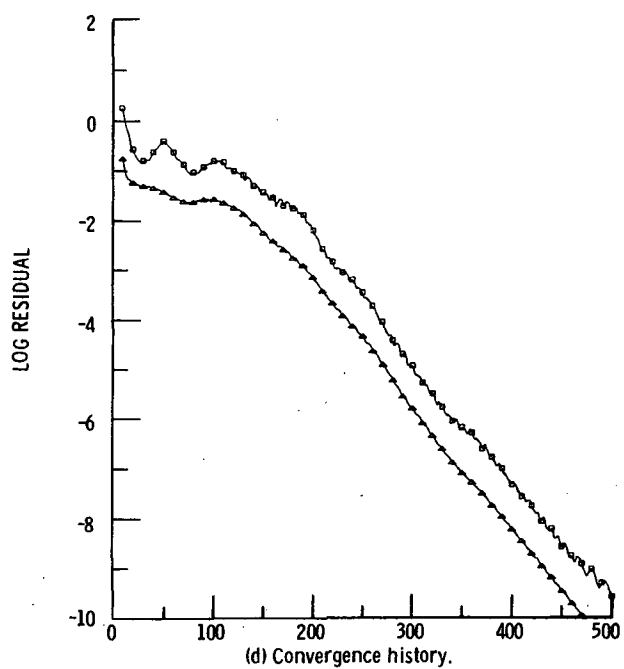
(a) Mach number contours.



(b) Mach number along the centerline.

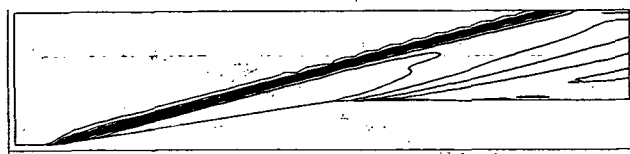


(c) Pressure along the centerline.

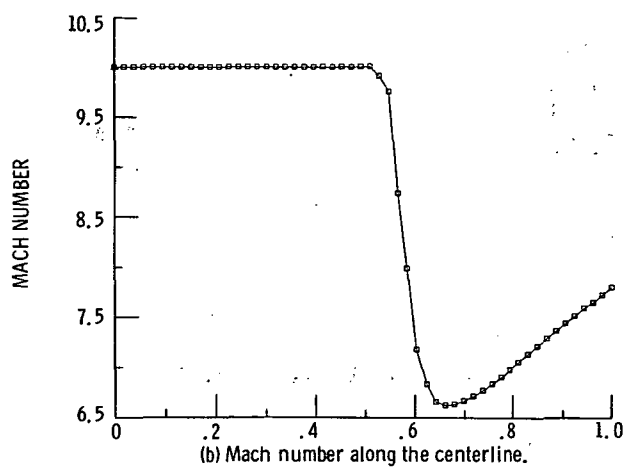


(d) Convergence history.

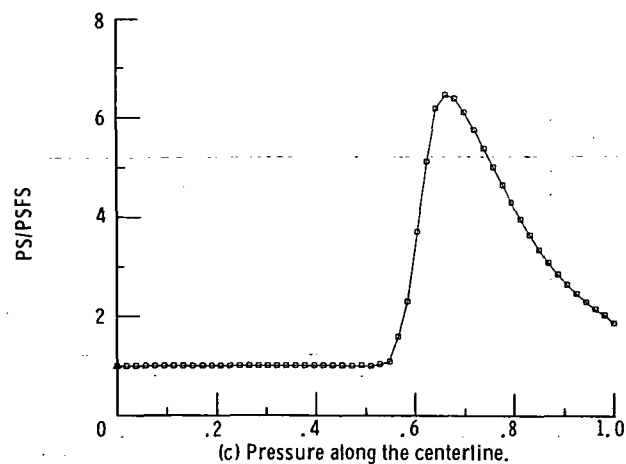
Figure 4. - Mach 5 inlet.



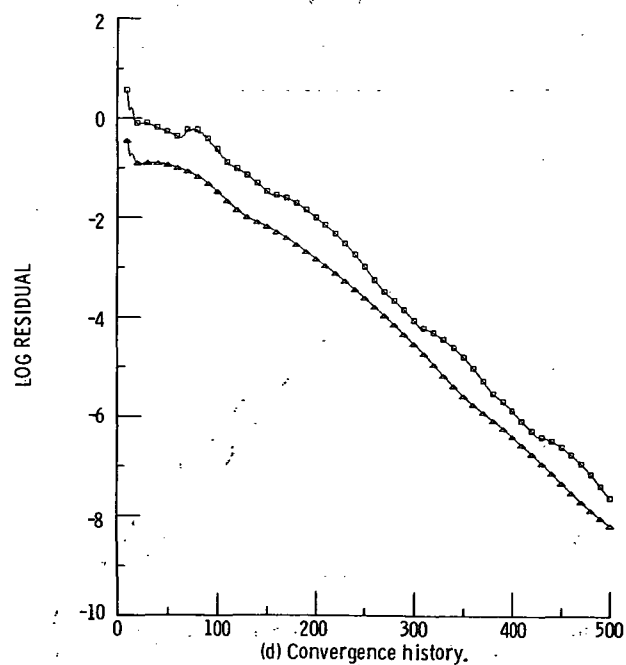
(a) Mach number contours.



(b) Mach number along the centerline.

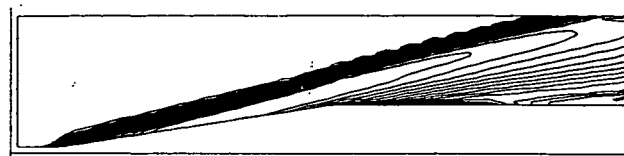


(c) Pressure along the centerline.

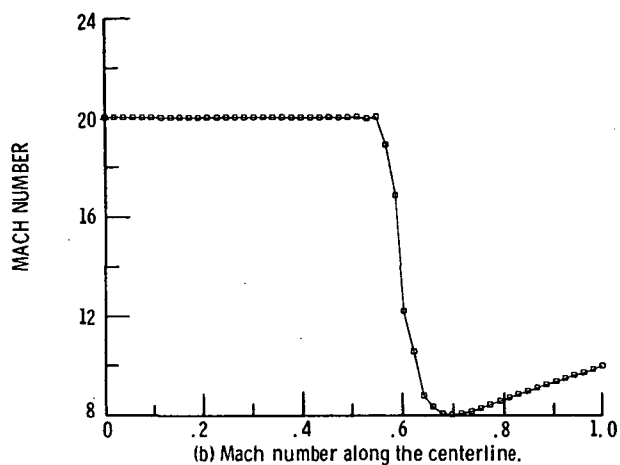


(d) Convergence history.

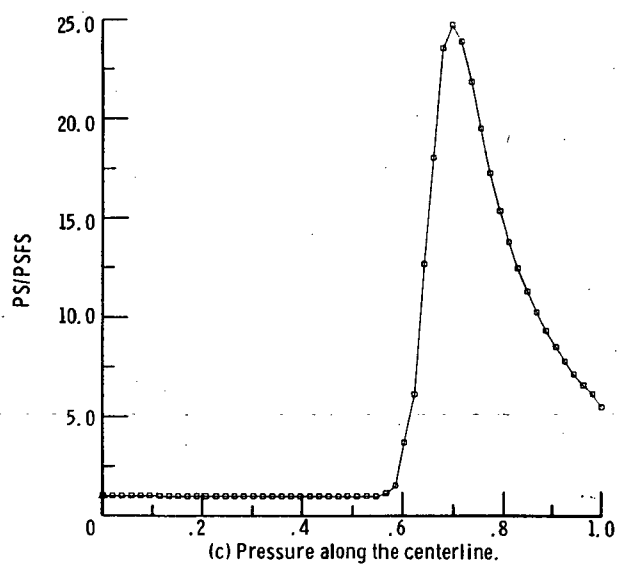
Figure 5. - Mach 10 inlet.



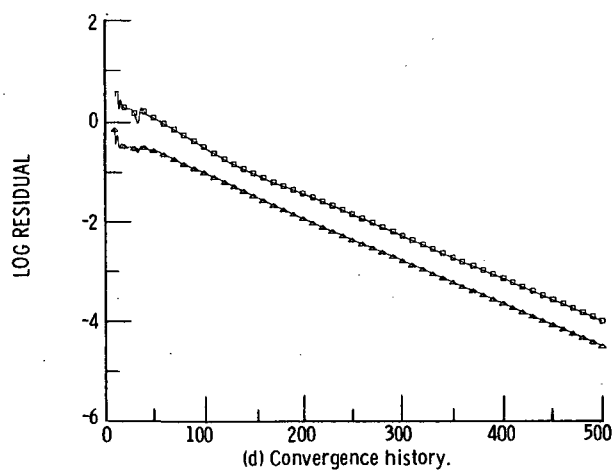
(a) Mach number contours.



(b) Mach number along the centerline.



(c) Pressure along the centerline.



(d) Convergence history.

Figure 6. - Mach 20 inlet.



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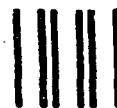
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